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# Sensitivity Study of Stochastic Walking Load Models

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## ABSTRACT

On flexible structures such as footbridges and long-span floors, walking loads may generate excessive structural vibrations and serviceability problems. The problem is increasing because of the growing tendency to employ long spans in structural design. In many design codes, the vibration serviceability limit state is assessed using a walking load model in which the walking parameters are modelled deterministically. However, the walking parameters are stochastic (for instance the weight of the pedestrian is not likely to be the same for every footbridge crossing), and a natural way forward is to employ a stochastic load model accounting for mean values and standard deviations for the walking load parameters, and to use this as a basis for estimation of structural response. This, however, requires decisions to be made in terms of statistical distributions and their parameters, and the paper investigates whether statistical distributions of bridge response are sensitive to some of the decisions made by the engineer doing the analyses. For the paper a selected part of potential influences are examined and footbridge responses are extracted using Monte-Carlo simulations and focus is on estimating vertical structural response to single person loading.

## NOMENCLATURE

| | | | | | |
|---|---|---|---|---|---|
| $f_0$ | Bridge frequency | $f_s$ | Step frequency | $f$ | Walking load |
| $l_s$ | Stride length | $m$ | Weight of pedestrian | $p$ | Prob. density function |
| $q$ | Modal load | $L$ | Bridge length | $M$ | Bridge modal mass |
| $P$ | Prob. distribution function | $a$ | Bridge acceleration | $\alpha$ | Dynamic load factor |
| $\zeta$ | Bridge damping ratio | $\mu$ | Mean value | $\sigma$ | Standard deviation |

## 1. INTRODUCTION

For the studies of this paper, walking-induced vibrations of footbridges are considered (but problems with walking-induced vibrations may also be encountered on flooring-systems). That vibration problems can occur in footbridges due to the action of walking is known to many after the closure of the London Millennium Bridge [1]. In codes of practise for footbridges it is often stated that the vibration serviceability limit state related to walking loads is to be checked. Generally, footbridge vibration problems may occur as a result of vertical or horizontal excitation. The present paper considers the vertical action.

As for the vertical action, most codes suggest to model it as a moving harmonic force. This approach is also taken in this paper, but whereas most current codes model the parameters of the harmonic force (amplitude and frequency) deterministically, the present paper models these parameters as random variables. This would seem sensible as research has documented that the excitation frequency (step frequency) is a random variable ([2,3]), in that the step frequency varies from one pedestrian to the next. Nevertheless, a number of current codes [4,5] suggests a load model in which the excitation frequency is to be set equal to the bridge frequency for computing bridge response. This approach definitely does not consider the probability of the modelled resonant excitation and the procedure does not provide information on the probability of reaching the calculated bridge vibration level; although this would seem to be valuable information for the bridge operator.

As for the amplitude of the harmonic force it is determined from pedestrian weight ($m$), and the dynamic load factor ($\alpha$). These are parameters which fundamentally are also stochastic, but in a number of codes these characteristics are modelled deterministically as well. For instance, some codes assume the weight of the pedestrian to be 75 kg.

An approach to a probability-based estimation of bridge vibration levels was introduced in [3], and this paper adapts the general idea behind it. It allows statistical distributions of bridge response (to actions of walking of a single pedestrian) to be determined, recognising that walking parameters are stochastic variables. However, in [3], the pedestrian weight ($m$) was modelled deterministically and a stochastic model for $m$ was not considered. For the studies of this paper an addition is made in which the pedestrian weight is modelled as a random variable (along with modelling the other parameters of the load model as random variables in the way suggested in [3]).

It is not quite obvious how to model the statistical distribution of pedestrian weight, but nevertheless it is one of the inputs needed for computing statistical distributions of bridge response. Generally not much information is available about weights of pedestrians, but a distribution type need be selected and then characteristics of the distribution are to be decided upon (such as mean value and standard deviation of the random variable).

In light of the fact that information available for the decisions is sparse, the paper approaches the problem the other way around. It seeks to examine how sensitive statistical distributions of bridge response are to choices made about distribution type, mean value and standard deviation of pedestrian weight; this in the hope that the findings in terms of sensitivity will be useful for the engineer making such decisions.

To facilitate the investigations a footbridge model is required, and to this end a pin-supported footbridge (idealised as a single-degree-of-freedom system) is employed. The response characteristic given focus is the midspan peak accelerations. For the sensitivity study, three different distribution types, mean values, and standard deviations for pedestrian weight are considered and used for calculating statistical distributions of footbridge response. For reference purposes a deterministic model for pedestrian weight is also employed.

The bridge excited by pedestrians in computations is introduced in section 2 along with the walking load model. Section 3 outlines study assumptions in terms of walking parameters, and section 4 describes how statistical distributions of bridge response are obtained. Section 5 presents results. The results are discussed, and a conclusion is provided.

## 2. MODEL OF BRIDGE AND BRIDGE EXCITATION

The modal characteristics of the bridge considered for the studies of this paper are shown in Table 1.

| $f_0$ | $M$ | $\zeta$ |
|---|---|---|
| 2.00 Hz | 39.500 kg | 0.3% |

**Table 1.** Dynamic characteristics.

The frequency of the bridge ($f_0$) is chosen such that it represents a bridge prone to react lively to actions of walking. As can be seen, the bridge damping ratio ($\zeta$) is quite low. The modal mass ($M$) is believed to be quite realistic considering the frequency of the bridge, as is the length of the bridge, $L$, which is assumed to be 43 m (between the two pin supports).

For the paper (and as often done for modelling the vertical excitation generated by a pedestrian [6,7]), the dynamic load acting on the bridge, $f(t)$, is modelled as shown in eq. 1 in which $t$ is time.

$$f(t) = mg\,\alpha\,\cos(2\pi f_s t) \tag{1}$$

It is a harmonic load with a frequency, $f_s$, representing the step frequency of walking. The step frequency is assumed constant during the locomotion of the pedestrian whilst crossing the bridge. This is an idealisation, but the paper considers that the value of $f_s$ will change from one pedestrian to the next in order to model randomness in the action of walking. This approach is also taken for the amplitude of the harmonic excitation, in that the dynamic load factor, $\alpha$, will be modelled as a random variable, as will the pedestrian weight, $m$ (in kg). The parameter $g$ represents acceleration of gravity. Generally, there would also be super-harmonics of the action of walking worth considering, but for the bridge considered in this paper it would be the first harmonic of the action (eq. 1) that is of interest (the bridge is modelled as a SDOF system and it is the first harmonic that can cause resonance as this is the load harmonic in close vicinity of the bridge frequency).

It can be shown that the modal load on the bridge (first vertical bending mode) may be computed using eq. 2:

$$q(t) = mg\,\alpha\,\cos(2\pi f_s t)\sin(\pi f_s l_s t/L) \tag{2}$$

In brief, this equation assumes that the mode space function of the first vertical bending mode of the bridge corresponds to a half-sine. Furthermore it assumes, that the pedestrian traverses the bridge with a locomotion style in which the stride length, $l_s$, (or step length) is constant. For the studies of this paper, randomness in stride length is considered (from one pedestrian to the next).

## 3. MODELS FOR WALKING PARAMETERS

For the studies of this paper primary focus is on various ways of modelling pedestrian weight, and implications hereof. The various assumptions made are outlined in section 3.1, and section 3.2 outlines study assumptions for other walking parameters.

### 3.1 Pedestrian weight

For pedestrian weight ($m$) three different stochastic models are assumed; a normal distribution, a log-normal distribution, and a uniform distribution. Mean values and standard deviations assumed for each of the distributions are listed in table 2.

| Variable | Unit | $\mu_m$ | $\sigma_m$ |
|---|---|---|---|
| $m$ | kg | 50 | $0.1n\mu$ ($n$ = 0,1,2,3) |
| | | 75 | $0.1n\mu$ ($n$ = 0,1,2,3) |
| | | 85 | $0.1n\mu$ ($n$ = 0,1,2,3) |

**Table 2.** Mean values $\mu$ and standard deviations $\sigma$

As it would appear, three different study assumptions are made for the mean value, and for each assumption in terms of mean value, four different assumptions are made for the standard deviation ($n$ = 0, 1, 2, 3). The value of $n$ equal to zero is the deterministic approach. In this model, the weight of all pedestrians crossing the bridge is assumed to equal the mean value defined for the population (e.g. 50 kg). For values of $n$ larger than zero, a stochastic model is assumed for pedestrian weight. As the value of $n$ increases (from 1 over 2 to 3), so does the standard deviation of pedestrian weight.

As an example, Figure 1 shows the normal, log-normal and uniform distribution functions of pedestrian weight when assuming $\mu_m$ = 75 kg and $\sigma_m$ = 0.1·2·75 kg = 15 kg (i.e. assuming $n$ = 2).

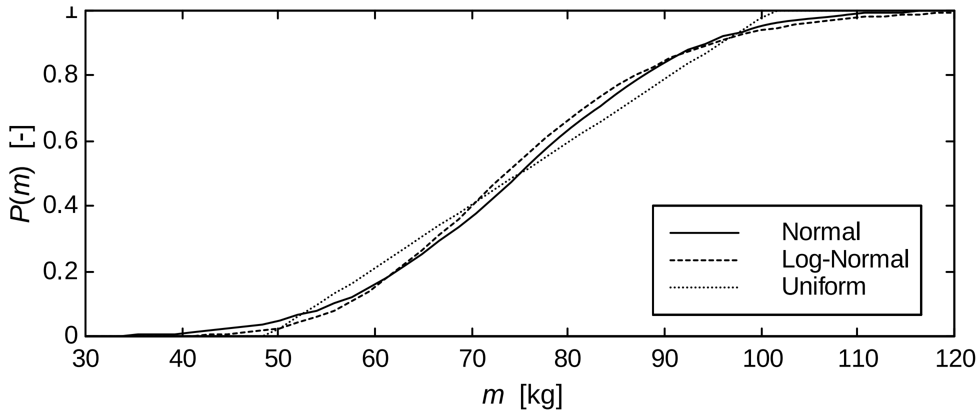


**Figure 1.** Probability distribution functions for *m*. Assumed is: $\mu_m$ = 75 kg and $\sigma_m$ = 15 kg.

As can be seen, the uniform distribution does not accept that pedestrian weight can be lower than, say 50 kg, and larger than, say 100 kg. Such restriction is not enforced for the normal and log-normal distributions.

### 3.2 Other walking parameters

For the dynamic load factor ($\alpha$), step frequency ($f_s$), and stride length ($l_s$) randomness is modelled. The study assumptions, covering mean values ($\mu$) and standard deviations ($\sigma$) for the individual random variables and associated distributions, are outlined in table 3. Table 3 represents stochastic models suggested in literature (references are stated) and they all rely on normal distributions.

| Variable | Unit | $\mu$ | $\sigma$ | Reference |
|---|---|---|---|---|
| $f_s$ | Hz | 1.99 | 0.173 | Matsumoto [2] |
| $\alpha$ | - | eq. 3 | $0.16\mu_\alpha$ | Kerr [9] |
| $l_s$ | m | 0.71 | 0.071 | Živanovic [3] |

**Table 3.** Mean value and standard deviation for $\alpha$, $f_s$, and $l_s$

In table 3 a reference is made to eq. 3, which describes the modelled relationship between the dynamic load factor (its mean value, $\mu_\alpha$) and the step frequency, $f_s$ (to be inserted in Hz):

$$\mu_\alpha = a\, f_s^3 + b\, f_s^2 + c\, f_s + d \tag{3}$$

where

$$a = -0.2649 \qquad b = 1.306 \qquad c = -1.7597 \qquad d = 0.7613 \tag{4}$$

Generally eq. 3 indicates that $\mu_\alpha$ is conditioned on $f_s$. The relationship is calibrated to measurement results in the frequency range 1 Hz < $f_s$ < 2.7 Hz.

Figure 2 illustrates some of the study assumptions defined in this section.

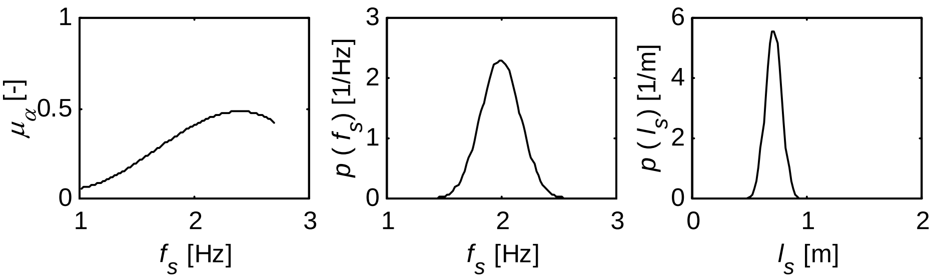


**Figure 2**. Relationship defined by eq. 3 (left), probability density function for step frequency, $p(f_s)$ (middle) and density function for stride length, $p(l_s)$ (right).

## 4. CHARACTERISTICS OF BRIDGE RESPONSE

For the calculations of bridge response, the bridge is assumed to be at rest when a pedestrian enters the bridge. The response considered is the vertical peak acceleration response encountered at midspan of the bridge, as this response characteristic is often used to evaluate the vibration serviceability limit state. Considering the various possible ways of modelling pedestrian weight interest is on results of characteristics of the statistical distribution of bridge peak response calculated on different assumptions. Here focus is on quantiles of bridge acceleration response, $a$. High quantiles, such as $a_{95}$ are believed to be of primary interest for the bridge designer and operator and the notation indicates that in 1 out 20 bridge crossings, the acceleration level $a_{95}$ is expected to be exceeded. For completeness, some other quantiles are also monitored.

The quantiles are extracted from statistical distributions of bridge response computed using MonteCarlo Simulation methods considering the modelled randomness in walking parameters. As many as 500,000 simulations runs (each emulating a pedestrian crossing the bridge) were made to provide confidence in the computed statistical distributions. For computing bridge response a Newmark time integration scheme was employed.

## 5. RESULTS

In terms of different quantiles of bridge acceleration response, the calculations gave the results shown in figure 3.

Focusing on the quantile $a_{75}$ (upper 3 plots) it appears that the mean value of pedestrian weight has a significant bearing on bridge response. An increase in mean value increases bridge loading and therefore its response. This is not surprising considering eq. 2. It can be shown that the calculated values of $a_{75}$ are close to linearly linked with the mean value of pedestrian weight ($\mu_m$) assumed for the calculations.

The upper 3 plots also reveal that for a given value of mean weight of the pedestrians (whether $\mu_m$ = 50, 75 or 85 kg), almost identical results in terms of $a_{75}$ are obtained whether a normal, a log-normal or a uniform distribution is assumed for pedestrian weight. The results also suggest that it is not important whether one or the other standard deviation is assumed ($n$ = 0, 1, 2, or 3). An almost identical result is obtained anyway. To give some perspective in terms of a realistic value of $n$ it might be 1.8. At least this was the value obtained by weighting a large number of students at Aalborg University. But in terms of an estimate of $a_{75}$ for the bridge considered in this paper, the value of $n$ appears to be of marginal importance. Basically, the deterministic model ($n$ = 0) would provide a sufficiently accurate estimate of $a_{75}$.

Turning to the quantiles $a_{95}$ and $a_{97.5}$ ($a_{97.5}$ especially), a tendency is seen in which an increase in standard deviation of pedestrian weight (increase in the value of $n$) results in gradually, but slightly, increasing values of the quantiles. This observation suggests that the random nature of pedestrian weight (specifically the standard deviation assumed) has some bearing on the uppermost quantiles of bridge acceleration response. This is observed regardless of whether a normal, log-normal or uniform distribution is assumed for pedestrian weight. In fact almost identical results in terms of $a_{95}$ and $a_{97.5}$ are obtained for the three different distributions. It is also seen that the value of $n$ (and thus the size of the standard deviation) only slightly influences, for instance, $a_{95}$.

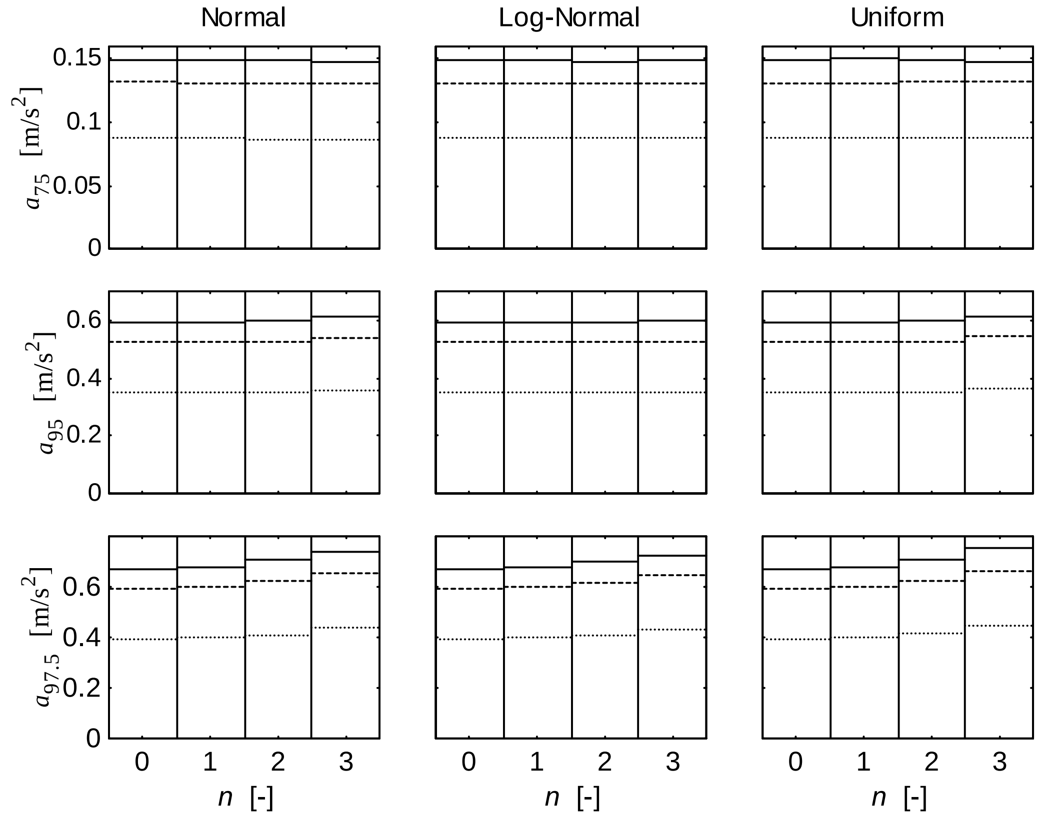


**Figure 3**. Three quantiles of bridge response and how they depend on $n$, $\mu_m$, and distribution type for $m$. Solid line: $\mu_m$ = 85 kg, Dashed line: $\mu_m$ = 75 kg, Dotted line: $\mu_m$ = 50 kg.

A different way of illustrating that the statistical distribution of bridge response only to some extent is influenced by the assumption of the size of the standard deviation is also illustrated in figure 4. The plot in figure 4 shows the probability distribution functions for bridge accelerations calculated assuming $n$ = 0 and $n$ = 3, respectively.

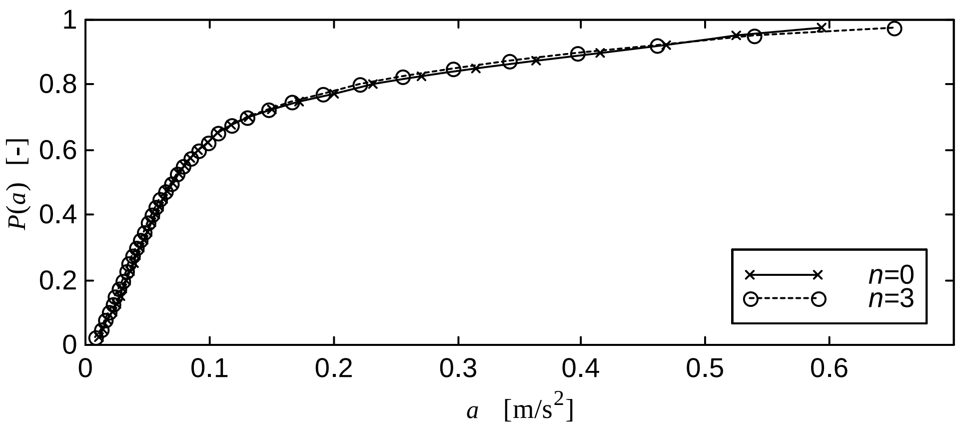


**Figure 4**. Probability distribution functions of bridge acceleration. $\mu_m$ = 75 kg and a normal distribution for $m$ is assumed (it is thus an example).

In figure 4 it can be seen that the highest standard deviation considered ($n$ = 3) result in a statistical distribution of bridge response which is fairly identical to the distribution calculated assuming $n$ = 0, which is the deterministic model for pedestrian weight. Minor differences can be noticed.

Figure 1 illustrated the three different distributions assumed for pedestrian weight (normal, log-normal, and uniform), and figur 5 illustrates the statistical distributions of bridge response calculated assuming the three different distributions for pedestrian weight.

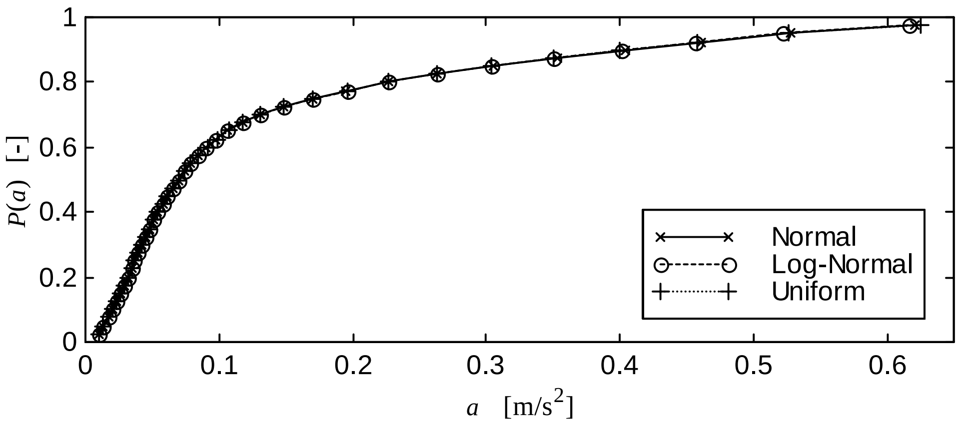


**Figure 5**. Probability distribution functions of bridge acceleration. Assumed is: $\mu_m$ = 75 kg and $\sigma_m$ = 15 kg (it is thus an example, as more combinations of $\mu_m$ and $\sigma_m$ have been investigated).

As can be seen, the output of calculations (the statistical distribution of bridge response calculated on three different assumptions about the distribution type for pedestrian weight) becomes almost identical even though the input for the calculations is in fact different.

**6. CONCLUSION AND DISCUSSION**

It was investigated how some of the choices that need be made, related to modelling pedestrian weight, influenced the statistical distribution of vertical bridge response for a particular footbridge. Three different statistical distributions were considered for pedestrian weight and for each distribution, different mean values and standard deviations were considered. It appeared that the statistical distribution of bridge response was not sensitive to whether a normal, log-normal, or uniform distribution was assumed for pedestrian weight. The results also showed that the statistical distribution of bridge response was very sensitive to the mean value of pedestrian weight but only slightly sensitive to the standard deviation of pedestrian weight applied for the calculations. A fully deterministic model of pedestrian weight (standard deviation set to zero) showed to provide a statistical distribution which only differed slightly from those calculated when pedestrian weight was modelled as a random variable.

This would immediately suggest that it might be unnecessary to model pedestrian weight as a random variable, but that it is quite important to employ a value of pedestrian weight that well represents the mean value of pedestrian weight found in the actual population of pedestrians expected to traverse the footbridge.

It should be recalled that these conclusions are reached studying only a single footbridge, and thus that they may not be valid for any footbridge. It is also important to recognise that the quite simplistic distributions of pedestrian weight employed for the studies of this paper might not be representative for actual populations of pedestrians. Not much data are available on pedestrian weight, but the paper may be considered a baseline study which can be extended considering other bridges and more complex distributions of pedestrian weight.

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